

Charmonium molecules?

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Abstract. In this talk we present some recent studies of multiquark components in the charmonium sector. We study the possible existence of compact four quark-states and meson-meson molecules in the charmonium spectroscopy.

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Since 2003 several states have been discovered in the charmonium mass region. While in the conventional description of the charmonium spectrum in terms of quark-antiquark pairs some states are still missing, the number of experimental states reported up to now is larger than empty spaces in the $c\bar{c}$ spectrum. This, together with other difficulties to explain observed states as simple quark-antiquark pairs triggered discussions on a possible exotic interpretation: four-quark states either as compact tetraquarks or slightly bound meson-meson molecules.

Some states recently found in the hidden-charm sector [1] may fit in the simple quark-model description as $(c\bar{c})$ pairs (i.e., $X(3940)$, $Y(3940)$, and $Z(3940)$ as radially excited χ_{c0} , χ_{c1} , and χ_{c2}), but others appear to be more elusive, in particular $X(3872)$, $Z(4430)^+$, and $Y(4140)$. The debate on the nature of these states, including the possibility of finding $c\bar{c}n\bar{n}$ four-quark states, is open with special emphasis on the $X(3872)$. Since it was first reported by Belle in 2003 [2], it has gradually become the flagship of the new armada of states whose properties make their identification as traditional $(c\bar{c})$ states unlikely. An average mass of 3871.2 ± 0.5 MeV and a narrow width of less than 2.3 MeV have been reported for the $X(3872)$. Note the vicinity of this state to the $D^0\bar{D}^{*0}$ threshold, $M(D^0\bar{D}^{*0}) = 3871.2 \pm 1.2$ MeV. With respect to the $X(3872)$ quantum numbers, although some caution is still required until better statistic is obtained [3], an isoscalar $J^{PC} = 1^{++}$ state seems to be the best candidate to describe the properties of the $X(3872)$.

In an attempt to disentangle the role played by multiquark configurations in the charmonium spectroscopy we have obtained an exact solution of the four-body problem based on an infinite expansion of the four-quark wave function in terms of hyperspherical harmonics (HH) [4]. The four-body Schrödinger equation has been solved accurately using two standard quark models containing a linear confinement supplemented by a Fermi–Breit one-gluon exchange interaction (BCN), and also boson exchanges between the light quarks (CQC). The model parameters were tuned in the meson and baryon spectra. The results are given in Table 1, indicating the quantum numbers of the state studied, J^{PC} , the maximum value of the grand angular momentum used in the HH expansion, K_{\max} , and the energy difference between the mass of the four-quark state, E_{4q} , and that of the lowest two-meson threshold calculated with the same potential model, Δ_E .

There are several issues that deserve our attention. First of all, the two-meson thresholds have been determined assuming quantum number conservation within exactly the same scheme used in the four-quark calculation. Dealing with strongly interacting particles, the two-meson states should have well defined total angular momentum, parity, and a properly symmetrized wave function if two identical mesons are considered (coupled scheme). When noncentral forces are not taken into account, orbital angular momentum and total spin are also good quantum numbers (uncoupled scheme). Although noncentral forces were not used, the coupled scheme is the relevant one for comparing with experimental data.

Secondly, it is particularly interesting to check the convergence of the method. We have plotted in Fig. 1 the energy of the $J^{PC} = 1^{++}$ state as a function of K . It can be observed how the BCN 1^{++} state does not converge to the lowest threshold for small values of K , being affected by the presence of an intermediate $J/\psi\omega|_S$ threshold with an energy of 3874 MeV. Once sufficiently large values of K are considered the system follows the usual convergence to the lowest threshold (see insert in Fig. 1). Thus, the method may find difficulties for states that are close below a two-meson threshold.

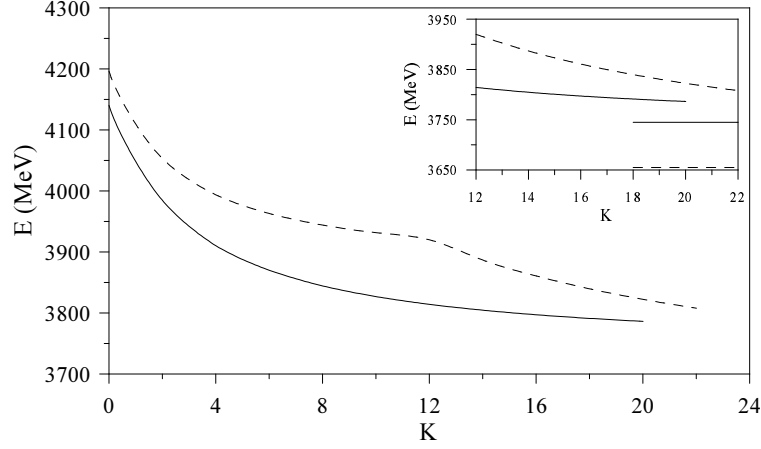


FIGURE 1. Energy of the 1^{++} state using the CQC (solid line) and BCN models (dashed line) as a function of K . The insert in the upper-right corner magnifies the large values of K to show the convergence to the corresponding threshold showed by a straight line.

Finally, besides trying to unravel the possible existence of bound $c\bar{c}n\bar{n}$ states one should aspire to understand whether it is possible to differentiate between compact and molecular states. A molecular state may be understood as a four-quark state containing a single physical two-meson component, i.e., a unique singlet-singlet component in the color wave function with well-defined spin and isospin quantum numbers. One could expect these states not being deeply bound and therefore having a size of the order of the two-meson system, i.e., $\Delta_R = R_{4q}/(r_{2q}^1 + r_{2q}^2) \sim 1$. Opposite to that, a compact state may be characterized by its involved structure on the color space, its wave function containing different singlet-singlet components with non negligible probabilities. One would expect such states would be smaller than typical two-meson systems, i.e., $\Delta_R < 1$. Let us notice that while $\Delta_R > 1$ but finite would correspond to a meson-meson molecule $\Delta_R \xrightarrow{K \rightarrow \infty} \infty$ would represent a two-meson threshold.

Considering these remarks and the results shown in Table 1 one can conclude that there are no deeply bound states (compact) in the $c\bar{c}n\bar{n}$ system. However, as explained above, being the HH method exact, it is not completely adequate to study states that are close to, but below, the charmed meson production threshold. Such states are called *molecular*, in the sense that they can be exactly expanded in terms of a single singlet-singlet color vector. Close to a threshold, methods based on a series expansion may fail to converge since arbitrary large number of terms are required to determine the wave function.

As mentioned above, the new experimental findings do not fit, in general, the simple predictions of the quark-antiquark schemes and, moreover, they overpopulate the expected number of states in (simple) two-body theories.

TABLE 1. $c\bar{c}n\bar{n}$ results.

$J^{PC}(K_{max})$	CQC		BCN	
	E_{4q}	Δ_E	E_{4q}	Δ_E
$0^{++}(24)$	3779	+34	3249	+75
$0^{+-}(22)$	4224	+64	3778	+140
$1^{++}(20)$	3786	+41	3808	+153
$1^{+-}(22)$	3728	+45	3319	+86
$2^{++}(26)$	3774	+29	3897	+23
$2^{+-}(28)$	4214	+54	4328	+32
$1^{-+}(19)$	3829	+84	3331	+157
$1^{--}(19)$	3969	+97	3732	+94
$0^{-+}(17)$	3839	+94	3760	+105
$0^{--}(17)$	3791	+108	3405	+172
$0^{--}(17)$	3791	+108	3405	+172
$2^{-+}(21)$	3820	+75	3929	+55
$2^{--}(21)$	4054	+52	4092	+52

This situation is not uncommon in particle physics. For example, in the light scalar-isoscalar meson sector hadronic molecules seem to be needed to explain the experimental data [5, 6, 7]. Also, the study of the NN system above the pion production threshold required new degrees of freedom to be incorporated in the theory, either as pions or as excited states of the nucleon, i.e., the Δ [8, 9]. This discussion suggests that charmonium spectroscopy could be rather simple below the threshold production of charmed mesons but much more complex above it. In particular, the coupling to the closest $(c\bar{c})(n\bar{n})$ system, referred to as *unquenching the naive quark model* [10], could be an important spectroscopic ingredient. Besides, hidden-charm four-quark states could explain the overpopulation of quark-antiquark theoretical states.

Trying to look for the possible existence of loosely bound molecular states close to a charmed meson threshold, we have used a different technique [11], we have solved the Lippmann-Schwinger equation looking for attractive channels that may contain a meson-meson molecule. In order to account for all basis states we allow for the coupling to charmonium-light two-meson systems.

When we consider the system of two mesons M_1 and \bar{M}_2 ($M_i = D, D^*$) in a relative S -state interacting through a potential V that contains a tensor force then, in general, there is a coupling to the $M_1\bar{M}_2$ D -wave and the Lippmann-Schwinger equation of the system is

$$t_{ji}^{\ell s \ell'' s''}(p, p''; E) = V_{ji}^{\ell s \ell'' s''}(p, p'') + \sum_{\ell'} \int_0^\infty p'^2 dp' V_{ji}^{\ell s \ell' s'}(p, p') \frac{1}{E - p'^2/2\mu + i\epsilon} t_{ji}^{\ell' s' \ell'' s''}(p', p''; E), \quad (1)$$

where t is the two-body amplitude, j, i , and E are the angular momentum, isospin and energy of the system, and $\ell s, \ell' s', \ell'' s''$ are the initial, intermediate, and final orbital angular momentum and spin; p and μ are the relative momentum and reduced mass of the two-body system, respectively. In the case of a two D meson system that can couple to a charmonium-light two-meson state, for example when $D\bar{D}^*$ is coupled to $J/\Psi\omega$, the Lippmann-Schwinger equation for $D\bar{D}^*$ scattering becomes

$$t_{\alpha\beta;ji}^{\ell_\alpha s_\alpha \ell_\beta s_\beta}(p_\alpha, p_\beta; E) = V_{\alpha\beta;ji}^{\ell_\alpha s_\alpha \ell_\beta s_\beta}(p_\alpha, p_\beta) + \sum_\gamma \sum_{\ell_\gamma} \int_0^\infty p_\gamma^2 dp_\gamma V_{\alpha\gamma;ji}^{\ell_\alpha s_\alpha \ell_\gamma s_\gamma}(p_\alpha, p_\gamma) \times G_\gamma(E; p_\gamma) t_{\gamma\beta;ji}^{\ell_\gamma s_\gamma \ell_\beta s_\beta}(p_\gamma, p_\beta; E), \quad (2)$$

with $\alpha, \beta, \gamma = D\bar{D}^*, J/\Psi\omega$.

We have consistently used the same interacting Hamiltonian to study the two- and four-quark systems to guarantee that thresholds and possible bound states are eigenstates of the same Hamiltonian. Such interaction contains a universal one-gluon exchange, confinement, and a chiral potential between light quarks [12]. We have solved the coupled channel problem of the $D\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$. In all cases we have included the coupling to the relevant $(c\bar{c})(n\bar{n})$ channel (from now on denoted as $J/\Psi\omega$ channels).

As we study systems with well-defined C -parity and since neither $D\bar{D}^*$ nor $\bar{D}D^*$ are eigenstates of C -parity, it is necessary to construct the proper linear combinations. Taking into account that $C(D) = \bar{D}$ and $C(D^*) = -\bar{D}^*$, it can be found that [13]:

$$D_1 = \frac{1}{\sqrt{2}}(D\bar{D}^* + \bar{D}D^*) \quad (3)$$

and

$$D_2 = \frac{1}{\sqrt{2}}(D\bar{D}^* - \bar{D}D^*) \quad (4)$$

are the eigenstates corresponding to $C = -1$ and $C = +1$, respectively.

Table 2 and Fig. 2 summarize our results. We have specified the quantum numbers of the attractive channels. The rest, not shown on the table, are either repulsive or have zero probability to contain a bound state or a resonance. Let us remark that, of all possible channels, only a few are attractive. Of the systems made of a particle and its corresponding antiparticle, the $J^{PC}(I) = 0^{++}(0)$ channel is always attractive. In general, the coupling to the $\eta_c\eta$ channel reduces the attraction, but there is still enough attraction to expect a resonance close and above the threshold. This channel is much more attractive for the $D^*\bar{D}^*$ system than for $D\bar{D}$, thus, in the latter one could expect a wider resonance. It is easy to explain the reason for such a close-to-bind situation with these quantum numbers. They can be reached from a two-meson system without explicit orbital angular momentum, while through a simple $c\bar{c}$ pair it needs a unit of orbital angular momentum. Similar arguments were used to explain the proliferation of light scalar-isoscalar mesons [5, 6, 7]. The most attractive channel in the $D\bar{D}^*$ case is the $J^{PC}(I) = 1^{++}(0)$ and can be explained as before, except

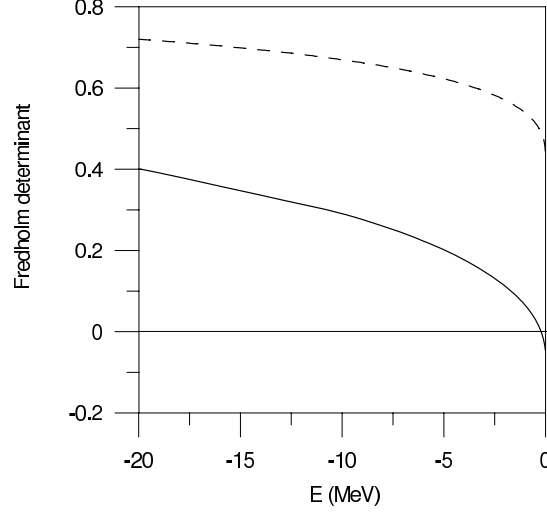


FIGURE 2. Fredholm determinant for the $J^{PC}(I) = 1^{++}(0) D\bar{D}^*$ system. Solid (dashed) line: results with (without) coupling to the $J/\Psi\omega$ channel.

the unity of intrinsic spin due to the D^* meson. A simple calculation of the $D\bar{D}^*$ system (Eq. (1)) indicates that the $J^{PC}(I) = 1^{++}(0)$ and $1^{+-}(1)$ are degenerate. It is the coupling to the $J/\Psi\omega$ (Eq. (2)) that breaks the degeneracy to make the $1^{++}(0)$ more attractive. The isospin 1 channel becomes repulsive due to the coupling to the lightest channel that includes a pion. Then, the existence of meson-meson molecules in the isospin one $D\bar{D}^*$ channels can be discarded. Using the coupling to the $J/\Psi\omega$, not present in the calculations at the hadronic level of [14, 15], we obtain a binding energy for the $J^{PC}(I) = 1^{++}(0)$ in the range 0 – 1 MeV, in good agreement with the experimental measurements of $X(3872)$ (see Fig. 2). This result supports the analysis of the Belle data on $B \rightarrow K + J/\Psi\pi^+\pi^-$ and $B \rightarrow K + D^0\bar{D}^0\pi^0$ that favors the $X(3872)$ being a bound state whose mass is below the $D^0\bar{D}^0$ threshold [13]. The existence of a bound state in the $1^{++}(0) D\bar{D}^*$ channel would not show up in the $D\bar{D}$ system because of quantum number conservation.

Finally, we have found that the $J^{PC}(I) = 2^{++}(0,1) D^*\bar{D}^*$ are also attractive due to the coupling to the $J/\Psi\omega$ and $J/\Psi\rho$ channels, respectively. This would give rise to new states around 4 GeV/ c^2 and one experimental candidate could be the $Y(4008)$. In this case, such a resonance would also appear in the $D\bar{D}$ system for large relative orbital angular momentum, $L = 2$. A similar behavior can be observed in resonances predicted for the $\Delta\Delta$ system [16].

In all cases, being loosely bound states whose masses are close to the sum of their constituent meson masses, their decay and production properties must be quite different from conventional $q\bar{q}$ mesons. Our calculation does not exclude a possible mixture of standard charmonium states in the channels where we have found attractive molecular systems. This admixture could explain some properties of the $X(3872)$ [17, 18]. We would like to emphasize the similarity of our results to those of Ref. [19] in spite of our different approach. Our treatment is general, dealing simultaneously with the two- and four-body problems and using an interaction containing gluon and quark exchanges instead of the simple two-body one-pion exchange potential of Ref. [19]. Nevertheless, we also concluded that the lighter meson-meson molecules are in the vector-vector and pseudoscalar-vector two-meson channels. Finally, let us remark that our

TABLE 2. Attractive channels for the two D -mesons system.

System	$J^{PC}(I)$
$D\bar{D}$	$0^{++}(0)$
$D\bar{D}^*$	$1^{++}(0)$
$D^*\bar{D}^*$	$0^{++}(0)$
$D^*\bar{D}^*$	$2^{++}(0)$
$D^*\bar{D}^*$	$2^{++}(1)$

approach could also be applied to the the $c\bar{c}s\bar{s}$ sector.

To summarize, our predictions show that no deeply bound states can be expected for the $c\bar{c}n\bar{n}$ system. Only a few channels can be expected to present observable resonances or slightly bound states. Among them, we have found that the $D\bar{D}^*$ system must show a bound state slightly below the threshold for charmed mesons production with quantum numbers $J^{PC}(I) = 1^{++}(0)$, that could correspond to the widely discussed $X(3872)$. Of the systems made of a particle and its corresponding antiparticle, $D\bar{D}$ and $D^*\bar{D}^*$, the $J^{PC}(I) = 0^{++}(0)$ is attractive. It would be the only candidate to accommodate a wide resonance for the $D\bar{D}$ system. For the $D^*\bar{D}^*$ the attraction is stronger and structures may be observed close and above the charmed meson production threshold. Also, we have shown that the $J^{PC}(I) = 2^{++}(0, 1)$ $D^*\bar{D}^*$ channels are attractive due to the coupling to the $J/\Psi\omega$ and $J/\Psi\rho$ channels. Due to heavy quark symmetry, replacing the charm quarks by bottom quarks decreases the kinetic energy without significantly changing the potential energy. In consequence, four-quark bottomonium mesons must also exist and have larger binding energies.

Particular analysis of different states based on different techniques have arrived to similar conclusions: Ref. [20], based on effective lagrangians, concludes that the $Y(3940)$ could be a $D^*\bar{D}^*$ $J^{PC}(I) = 0^{++}(0)$ or $2^{++}(0)$ meson-meson molecule and the $Y(4140)$ could be a $D_s^*\bar{D}_s^*$ $J^{PC}(I) = 0^{++}(0)$ or $2^{++}(0)$ meson-meson molecule; Ref. [21], based on dynamically generated resonances, concludes that the $Y(3940)$, $Z(3940)$ and $X(4160)$ could be $D^*\bar{D}^*$ and $D_s^*\bar{D}_s^*$ $0^{++}(0)$ and $2^{++}(0)$ states; Ref. [22], based on QCD sum rules, concludes that the $Y(4140)$ could be a $D_s^*\bar{D}_s^*$ $J^{PC}(I) = 0^{++}(0)$ or $2^{++}(0)$ meson-meson molecule; Ref. [23], based on a one-boson exchange model, concludes that the $Y(4140)$ could be a $D_s^*\bar{D}_s^*$ $J^{PC}(I) = 0^{++}(0)$ meson-meson molecule.

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REFERENCES

1. J. L. Rosner, *J. Phys. Conf. Ser.* **69**, 012002 (2007).
2. Belle Collaboration, S. -K. Choi et al., *Phys. Rev. Lett.* **91**, 262001 (2003).
3. K. K. Seth, *AIP Conf. Proc.* **814**, 13 (2006).
4. J. Vijande, E. Weissman, N. Barnea, and A. Valcarce, *Phys. Rev. D* **76**, 094022 (2007).
5. R. L. Jaffe, *Phys. Rev. D* **15**, 267 (1977).
6. J. D. Weinstein and N. Isgur, *Phys. Rev. D* **41**, 2236 (1990).
7. G. 't Hooft, G. Isidori, L. Maiani, A. D. Polosa, and V. Riquer, *Phys. Lett. B* **662**, 424 (2008).
8. C. Elster, K. Holinde, D. Schutte, and R. Machleidt, *Phys. Rev. C* **38**, 1828 (1988).
9. H. Pöpping, P. U. Sauer, and X. -Z. Zhang, *Nucl. Phys. A* **474**, 557 (1987).
10. F. E. Close, *arXiv:0706.2709*.
11. T. Fernández-Caramés, A. Valcarce, and J. Vijande, *Phys. Rev. Lett.* **103**, 222001 (2009).
12. J. Vijande, F. Fernández, and A. Valcarce, *J. Phys. G* **31**, 481 (2005).
13. E. Braaten and M. Lu, *Phys. Rev. D* **76**, 094028 (2007).
14. Y. -R. Liu, X. Liu, W. -Z. Deng, and S. -L. Zhu, *Eur. Phys. J. C* **56**, 63 (2008).
15. C. E. Thomas and F. E. Close, *Phys. Rev. D* **78**, 034007 (2008).
16. A. Valcarce, H. Garcilazo, R. D. Mota, and F. Fernández, *J. Phys. G* **27**, L1 (2001).
17. S. S. Gershtein, A. K. Likhoded, and A. V. Luchinsky, *Phys. Rev. D* **74**, 016002 (2006).
18. C. Bignamini, B. Grinstein, F. Piccinini, A. D. Polosa, and C. Sabelli, *Phys. Rev. Lett.* **103**, 162001 (2009).
19. N. A. Törnqvist, *Phys. Rev. Lett.* **67**, 556 (1991).
20. T. Branz, T. Gutsche, and V. E. Lyubovitskij, *Phys. Rev. D* **80**, 054019 (2009) and these proceedings.
21. R. Molina and E. Oset, *Phys. Rev. D* **80**, 114013 (2009) and these proceedings.
22. R. M. Albuquerque, M. E. Bracco, and M. Nielsen, *Phys. Lett. B* **678**, 186 (2009).
23. G. -J. Ding, *Eur. Phys. J. C* **64**, 297 (2009).